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Weibel instability in Weakly Relativistic Laser Fusion Plasma

Slimen. Belghit*, Beddiaf. Zaidi, Oussama. Boultif and Abdelaziz. Sid
PRIMALAB Laboratory, Department of Physics, Faculty of Material Sciences, University of Batna1, Batna, *05000DZ,* Algeria.

*Belghit.slimen@gmail.com

ABSTRACT

In this work, the Weibel instability due to inverse bremsstrahlung (IB) absorption in laser fusion plasma has been investigated. The stabilization effect due to the coupling of the self-generated magnetic field by Weibel instability with the laser wave field is explicitly showed. In this study, the relativistic effects are taken into account; here the basic equation is the relativistic Fokker-Planck equation. The main obtained result is that the coupling of self-generated magnetic field with the laser wave causes a stabilizing effect of excited Weibel modes. We found a decrease in the spectral range of Weibel unstable modes. This decreasing is accompanied by a reduction of two orders in the growth rate spectrum of instability, or even stabilization of these modes. It has been shown that the previous analysis of the Weibel instability due to IB have overestimated the values of the generated magnetic fields.

Keywords: relativistic Weibel instability, laser fusion plasma, static magnetic field, stabilization, Relativistic laser plasma interaction.

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INTRODUCTION

Weibel instability **[1]** is a micro instability. It corresponds to the excitation of electromagnetic modes in plasmas characterized by temperature anisotropy. In a microscopic way, this corresponds to plasma described by an anisotropic distribution function in velocity space. The temperature anisotropy can be generated in plasma by different mechanisms, specifically the heat transport, the expansion of the plasma, and the inverse bremsstrahlung absorption **[2]**. We aim in this work to investigate the Weibel instability due to inverse bremsstrahlung absorption taking into account the stabilization effect due to the coupling of the self-generated magnetic field by the Weibel instability with the laser wave field in the relativistic regime, this needs to derive the dispersion relation of low-frequency

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electromagnetic Weibel modes in plasma heated by a laser pulse. The basic equation in this investigation is the relativistic Fokker-Planck equation **[3]**. It results highlight new terms in the dispersion relation due to the coupling between the laser electric field, and the resulting magnetic field by the Weibel instability. These terms contribute to the instability and the convection of Weibel modes. We consider inhomogeneous plasma in interaction with a high frequency and low magnitude laser field. We calculate the distribution function from the anisotropic Fokker-Planck equation. For this we use the method of separation of time scales and the iterative method. After, we solve the linear part of the Fokker-Planck equation associated with the disruption of the distribution function and establish the dispersion relation of the Weibel modes. Solving the dispersion relation leads to the calculation of the instability growth rate.

The present work is organized as follows: in section 1, we present the basic equation used in our theoretical model which is the Fokker-Planck equation. In section 2, we calculate the high frequency distribution function. In section 3, we calculate the low frequency distribution function. In section 4, we present an analysis of Weibel instability. Finally in section 5 we present a discussion of results and a brief conclusion summarizing our main results is given.

1- Basic equation

To describe fully ionized plasma where interactions between particles are dominated by the Coulomb interactions, it is judicious to use the Fokker-Planck equation given in the Ref **[4]**. For electrons, it is written in the laboratory frame as:

$$\frac{\partial f}{\partial t}+\frac{\vec{p}}{m_e\gamma_l}.\frac{\partial f}{\partial \vec{r}}-e\left(\vec{E}+\frac{\vec{p}}{m_e\gamma_l}\times\vec{B}\right).\frac{\partial f}{\partial \vec{p}}=C_{ei}(f)+C_{ee}(f) \tag{1}$$

Where $f=f(\vec{r},\vec{p},t)$ is the electrons distribution functions, $\gamma_l=(1+\frac{p^2}{m_e^2c^2})^{1/2}$ is the relativistic Lorentz factor, $p=m\,\gamma_l v$ is the quantity of movement, m_e is the electron mass and e is the elementary charge.

$C_{ee}(f)$ and $C_{ei}(f)$ mean respectively is the electron-electron and electron-ion collision **[5]**.

$\vec{E}$ and $\vec{B}$ are respectively the electric and the magnetic fields present in the plasma.written as: $\vec{E}=\vec{E}_h+\vec{E}_s$ and $\vec{B}=\vec{B}_h+\vec{B}_s$, where $\vec{E}_h$ and $\vec{B}_h$ represent the high-frequency fields associated to the laser wave, $\vec{E}_s$ and $\vec{B}_s$ mean low frequency fields associated to the disturbance in the plasma. The contribution of the high-frequency laser wave magnetic field $\vec{B}_h$, can be neglected compared to the contribution of the laser wave high-frequency electric field, $\vec{E}_h$ as typically: $\vec{E}_h/\vec{B}_h \sim c$. and $\vec{E}_s/\vec{B}_s \sim \frac{\omega}{k} \ll 1$, The temporal dependence of $\vec{E}_h$ is supposed to be a normal mode:

$$\vec{E}_h = \vec{E}_0 Re[\exp(i\omega_l t)] \tag{2}$$

where $\vec{E}_0$ and ω_l are respectively the complex magnitude and the frequency of the laser wave.

In order to solve the equation (1) we consider two time scales, a low-frequency hydrodynamic time scale and high-frequency (laser field) one. Therefore, the electronic distribution function f can be written as the sum of a quasi-static distribution function f_s , which varies slowly over time and a high-frequency distribution function f_h , which follows the temporal variation of high frequency laser electric field $\vec{E}_h$, so:

$$f(\vec{r},\vec{p},t) = f_s(\vec{r},\vec{p},t) + Re(f_h(\vec{p})\exp(i\omega t)) \tag{3}$$

Note that the indices "s" and "h" refer the time scales (low frequency) and high frequency respectively and will be used throughout this work.

The separation of time scales in equation (1) leads to two kinetic equations: a quasi-static kinetic equations and a high-frequency kinetic equation, so:

$$\frac{\partial f_h}{\partial t} - e\vec{E}_h.\frac{\partial f_s}{\partial \vec{p}} = e\left(\vec{E}_s + \frac{\vec{p}}{m_e\gamma_l} \times \vec{B}_s\right).\frac{\partial f_h}{\partial \vec{p}} + C_{ei}(f_h) \tag{4}$$

$$\frac{\partial f_s}{\partial t}+\frac{\vec{p}}{m_e\gamma_l}.\frac{\partial f_s}{\partial \vec{r}}-e\left(\vec{E}_s+\frac{\vec{p}}{m_e\gamma_l}\times\vec{B}_s\right).\frac{\partial f_s}{\partial \vec{p}}-C_{ei}(f_s)$$
$$=\langle e\vec{E}_h.\frac{\partial f_h}{\partial \vec{p}}\rangle \qquad (5)$$

where The symbol < > denotes the average over the laser wave cycle time $T=2\pi/\omega_l$. The above two coupled equations are the basic equations in the present work. Note here that the terms in the electric field $\vec{E}_s$ and magnetic field, $\vec{B}_s$ reflect the inclusion, in our study, of the low frequency electromagnetic field. In particular, the first term on the right hand side in equation (4) reflects the coupling of quasi-static fields with the laser field. Let us remember here that this field present in the plasma is generated by the mechanism of the Weibel instability. The right-hand side of equation (5) is a term of beat or inverse bremsstrahlung; which translates the contribution of the laser field in the description of the distribution function f_s.

2- Calculation of high frequency distribution function.

For computing the high frequency distribution function from equation (4), we suppose that the effect of the quasi-static field and the (e-i) collisions are small compared to the effect of the high frequency laser field. Then it is judicious to consider the following scaling for the high frequency distribution function:

$$f_h=f_h^{(0)}(\omega_l)+f_h^{(1)}\left(\frac{v_{ei}}{\omega_l},\frac{\omega_c}{\omega_l}\right) \qquad (6)$$

where the index 0 and 1 correspond to the magnitude order of high frequency distribution function.

First we analyze the equation (6) for typical physical parameters of laser-plasma interactions: electronic temperature, $T_e=10KeV$, the (e-i) mean free path, $\lambda_{ei}=1\mu m$ and laser wave length $\lambda_l=1.06\mu m$. It appears that the laser frequency ω_l, is very important that the collisions frequency, v_{ei} . At the zero order, equation (4) is written as:

$$\frac{\partial f_h}{\partial t} = e\vec{E}_h.\frac{\partial f_s}{\partial \vec{p}} \qquad (7)$$

where the f_h solution is:

$$f_h^{(0)} =-\frac{ie}{\omega_l}E_h.\frac{\partial f_s}{\partial \vec{p}} \qquad (8)$$

By using iterative method, is calculated as:

$$f_h =-\frac{-ie}{\omega_l}E_{hj}.\frac{\partial f_s}{\partial p_j}-\frac{eC_{ei}(f_s)}{\omega^2}E_{hj}.\frac{\partial f_s}{\partial p_j} -\frac{e^2}{\omega_l^2}E_{hj}\left(E_s+\frac{\vec{p}}{m_e\gamma_l}\times\vec{B}_s\right)\frac{\partial}{\partial p_k}\left(\frac{\partial f_s}{\partial p_j}\right) \qquad (9)$$

where we have used the Einstein notation on the repeated index k and j .

3- Calculation of low frequency distribution function

In order to obtain the low frequency distribution function, we substitute the expression of the high frequency distribution function (eq. 9) in equation (5). After some mathematical investigations, the following equation is obtained:

$$\frac{\partial f_s}{\partial t}+\frac{\vec{p}}{m_e\gamma_l}.\frac{\partial f_s}{\partial \vec{r}}-e\left(\vec{E}_s+\frac{\vec{p}}{m_e\gamma_l}\times\vec{B}_s\right).\frac{\partial f_s}{\partial \vec{p}}-C_{ei}(f_s) = S_{IB} \qquad (10)$$

Where

$$S_{IB} =-\frac{1}{2}\left(\frac{e}{\omega_L}\right)^2 E_{hl}E_{hj}E_{si}\frac{\partial}{\partial p_l}\left(\frac{\partial^2 f_s}{\partial p_i \partial p_j}\right)-\frac{e}{2}\left(\frac{e}{\omega_L}\right)^2\frac{\partial}{\partial p_l}E_{hl}E_{hj}\left(\frac{\vec{p}}{m_e\gamma_l}\times\vec{B}_s\right)\frac{\partial}{\partial p_i}\left(\frac{\partial f_s}{\partial p_j}\right) -\frac{1}{2}\left(\frac{e}{\omega_L}\right)^2 E_{hl}E_{hj}\frac{\partial}{\partial p_l}\left(C_{ei}(p)\frac{\partial f_s}{\partial p_j}\right) \qquad (11)$$

For resolve this equation, we consider the following scaling:

$$f_s = f_s^{(0)} + \delta f_s \qquad (12)$$

with $\delta f \ll f_s^{(0)}$,The distribution function $f_s^{(0)}$ describes the plasma in presence of high frequency laser field E_h, however δf corresponds to the perturbation associated to quasi-static electromagnetic field: E_s and B_s. Using the above development (11), The evolution equation of the perturbed function is obtained from low frequency equation (10) by considering the first term order, so:

$$\frac{\partial \delta f_s}{\partial t} + \frac{\vec{p}}{m_e \gamma_l} . \frac{\partial \delta f_s}{\partial \vec{r}} - e\left(\vec{E}_s + \frac{\vec{p}}{m_e \gamma_l} \times \vec{B}_s\right) . \frac{\partial f_s^{(0)}}{\partial \vec{p}} - C_{ei}(p)\delta f_s = S_{IB}(\delta f_s) + S_{IB}\left(f_s^{(0)}\right) \qquad (13)$$

Where

$$S_{IB}(\delta f_s) =- \frac{1}{2} p_0^2 \frac{\partial}{\partial p_x}\left[C_{ei}(p) \frac{\partial \delta f_s}{\partial p_x}\right] \qquad (14)$$

and

$$S_{IB}\left(f_s^{(0)}\right) =- \frac{e}{2} p_0^2 E_{si} \frac{\partial}{\partial p_x}\left[\frac{\partial^2 f_s^{(0)}}{\partial p_i \partial p_x}\right] - \frac{e}{2} p_0^2 \frac{\partial}{\partial p_x}\left(\frac{\vec{p}}{m_e \gamma_l} \times \vec{B}_s\right) \frac{\partial^2 f_s^{(0)}}{\partial p_i \partial p_x} \qquad (15)$$

We now return to the previous analysis of Weibel modes by considering the electron ion collisions as described by the relaxation operator of Krook type, as follow:

$$C_{ei}(f_l) = \frac{-\upsilon}{p^3} m_e^3 \gamma_l^3 \, l(l+1)[f_l - f_h] \qquad (16)$$

Where $v = \frac{p_t^4}{2\lambda_{ei}}$ and λ_{ei} is the e-i mean free path. and l is the order of the Legender polynomial or the ordre of the component of the distribution function trucated on the Legender polynomials.

3-1-Calculation of the distribution function order (0)

Using the above development (12) and (16), we obtain the equation of order 0 as:

$$\frac{\partial f_s^{(0)}}{\partial t}+\frac{1}{2}\left(\frac{e}{\omega_L}\right)^2 E_{hj}E_{hl}\cdot\frac{\partial}{\partial p_l}\left(C_{ei}(p)\frac{\partial f_s^{(0)}}{\partial p_j}\right)=C_{ei}(p)f_s^{(0)} \qquad (17)$$

In the subsequent, we suppose that the non-perturbed plasma is homogenous in presence of a high frequency laser electric field, $\vec{E}_h$, with a linear polarization on the x direction. In this geometry the above equation becomes:

$$\frac{\partial f_s^{(0)}}{\partial t}+\frac{1}{2}p_0^2\cdot\frac{\partial}{\partial p_l}\left(C_{ei}(p)\frac{\partial}{\partial p_j}f_s^{(0)}\right)=C_{ei}(p)f_s^{(0)} \qquad (18)$$

Where $p_0=\frac{eE_{hl}}{\omega}$ is the electrons momentum oscillation in the laser electric field, $\vec{E}_h$ The electrons oscillation induces an anisotropic distribution function in the direction of $\vec{E}_h$: $f_s^{(0)}(\vec{p},t)=f_s^{(0)}(p,p_x,t)$. With introduction of the variable, $\mu=\frac{p_x}{p}$ the above equation (18) is presented as:

$$\frac{\partial f_{sl}^{(0)}}{\partial t}+\frac{C_{ei}}{2}p_0^2\left[\mu^2\frac{\partial^2 f_{sl}^{(0)}}{\partial p^2}+\frac{2\mu}{p}(1-\mu^2)\frac{\partial^2 f_{sl}^{(0)}}{\partial p\delta\mu}-\frac{6\mu}{p^2}(1-\mu^2)\frac{\partial f_{sl}^{(0)}}{\partial\mu}+\frac{1}{p}(1-4\mu^2)\frac{\partial f_{sl}^{(0)}}{\partial p}+\frac{1}{p^2}(1-\mu^2)^2\frac{\partial^2 f_{sl}^{(0)}}{\partial\mu^2}+3\mu^2\frac{p}{m^2c^2}\frac{1}{\gamma_l^2}\frac{\partial f_{sl}^{(0)}}{\partial p}+3\mu(1-\mu^2)\frac{1}{m^2c^2}\frac{1}{\gamma_l^2}\frac{\partial f_{sl}^{(0)}}{\partial\mu}\right]=C_{ei}(f_{S0}) \qquad (19)$$

Where Now, we develop the distribution function $f_s^{(0)}(p,\mu)$. on the Legendre polynomials $p_l(\mu)$, **[6,7]**, Then the above equation (19) can be developed After some algebra using recurrence relations between Legendre polynomials of several orders , we find the equation of the isotropic distribution function, which corresponds to the projection of the above equation (19) on the Legendre polynomial of order 0,so:

$$\frac{\partial f_{s0}^{(0)}}{\partial t}+C_{ei}\frac{p_0^2}{2}\left[\frac{1}{3}p\frac{\partial}{\partial p}\left(\frac{1}{p^4}\frac{\partial f_{s0}^{(0)}}{\partial p}\right)+\frac{1}{m^2c^2}\frac{1}{\gamma_l^2}\frac{1}{p^2}\frac{\partial f_{s0}^{(0)}}{\partial p}\right]=C_{ei}\left(f_{s0}^{(0)}\right) \qquad (20)$$

In this equation, the terms proportional to the second anisotropic distribution function $f_{s2}^{(0)}$, are ignored. This is justified by the fact that: $\frac{f_{s2}^{(0)}}{f_{s0}^{(0)}}\sim\frac{p_0^2}{p_t{}^2}\ll 1$, which correspond to the low magnitude laser wave approximation largely fulfilled in the laser-plasma interaction experiments. Note that this equation corresponds to that obtained in the reference **[8]**.

The equation of the second anisotropic, $f_{s2}^{(0)}$, is calculated under the same approximation by projection of the equation (18) on Legendre polynomial, p_2, so:

$$\frac{\partial f_{s2}^{(0)}}{\partial t}+\frac{\nu m_e^3\gamma_l^3}{p^3}\frac{p_0^2}{3\sqrt{5}}\left[\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\frac{\partial f_{s0}^{(0)}}{\partial p}\right)+\frac{3}{m^2c^2}\frac{1}{\gamma_l^2}\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^3 f_{s0}^{(0)}\right)\right]=\frac{\nu m_e^3\gamma_l^3}{p^3}f_{s2}^{(0)} \qquad (21)$$

where we have neglected in this equation the terms proportional to $f_{s4}^{(0)}$ and those proportional to $\frac{p_0^2}{p_t{}^2}.f_{s2}^{(0)}$ Then, the expression of the second anisotropic distribution function is obtained in the stationary approximation is $\frac{\partial f_{s2}^{(0)}}{\partial t}\approx 0$, generally considered in the Weibel instability analysis, so:

$$f_{s2}^{(0)}=\frac{p_0^2}{3\sqrt{5}}\left[\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\frac{\partial f_{s0}^{(0)}}{\partial p}\right)+\frac{3}{m^2c^2}\frac{1}{\gamma_l^2}\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^3 f_{s0}^{(0)}\right)\right] \qquad (22)$$

3-2-Calculation of the perturbed distribution function.

it appears that the perturbed distribution function δf_s , depends on the non perturbed distribution function, $f_{s0}^{(0)}$. Some simplification can be made on equation (13). In fact the term $\frac{\partial\delta f_s}{\partial t}\sim\omega\,\delta f_s$ (where ω is the Weibel mode pulsation), can be neglected compared to the collisions term, $\sim\nu_{ei}$,that the interested Weibel modes are quasi-statics: $\omega\ll\nu_{ei}$. Furthermore, by considering that $\frac{p_0^2}{p_t{}^2}\ll 1$, which corresponds to a low magnitude laser wave, it appears also that $S_{BI}(\delta f_s)$ is small compared to $\nu_{ei}\,\delta f_s$. Finally, by considering the Faraday law, $E_s=\frac{\omega}{k}B_s$, δf_s and the condition, $\frac{\omega}{kp_t}\ll 1$, the electric fied term can be neglected compared to magnetic field term in the expression of δf_s . With these approximations,

The linear polarization of the laser wave in the x direction allows to an positive anisotropy in temperature: $T_X>T_\perp$. This is susceptible to excite perpendicular Weibel modes: $\vec{k}\perp\vec{x}$ For simplification, we consider the geometry: ($\overrightarrow{E_s}\|\overrightarrow{E_h}\|\vec{x}, B_s\|\vec{y}, \vec{k}\|\vec{z}$). In this geometry, the equation (13) can be presented as:

$$ik\frac{p_z}{m_e\gamma_l}.\delta f_s - C_{ei}(p)\delta f_s$$
$$= eE_s\frac{\partial f_s^{(0)}}{\partial p_x} - \frac{eB_s}{m_e\gamma_l}\left(p_z\frac{\partial(p_2 f_{s2}^{(0)})}{\partial p_x} - p_x\frac{\partial(p_2 f_{s2}^{(0)})}{\partial p_z}\right) + S_{BI}\left(f_{s0}^{(0)}\right) \quad (23)$$

where

$$S_{BI}\left(f_{s0}^{(0)}\right) = -\frac{1}{2}\frac{eB_s}{m_e\gamma_l}p_0^2\left(p_x\frac{\partial^2 f_{s0}^{(0)}}{\partial p_x\partial p_z} - p_z\frac{\partial^2 f_{s0}^{(0)}}{\partial p_x^2}\right) \quad (24)$$

In the above equation (23), the anisotropic component has been neglected in the term of E_s. In the right hand side of equation (23), the first term is the source term of Weibel instability, however the term $S_{BI}\left(f_{s0}^{(0)}\right)$ corresponds to the coupling between the quasi-static magnetic field, B_s , and the laser wave field *($\sim p_0$)*. This term has been ignored in the previous studies of the Weibel instability **[2,9]** in spite that it is comparable to the source term. .For resolve equation (23), first we express the velocity vector in spherical coordinates:

$p_z = pcos\theta$,$p_x = psin\theta cos\varphi$,$p_y = psin\theta sin\varphi$.where equation (23) can be presented as:

$$\left(ik\frac{p_z}{m_e\gamma_l}cos\,\theta - C_{ei}(p)\right)\delta f_s = S_E + S_B \quad (25)$$

where

$$S_E = \ eE_s sin\theta cos\varphi\ \frac{\partial f_{s0}^{(0)}}{\partial p} \quad (26)$$

$$S_B = -\ e\frac{eB_s}{\gamma_l m_e}cos\theta sin\theta cos\varphi\left\{\frac{1}{p^3}\frac{\partial}{\partial p}\left(p^3\frac{\partial f_{s0}^{(0)}}{\partial p}\right) + \frac{1}{m_e^2c^2\gamma_l^2}(9f_{s0}^{(0)} + \frac{7}{2}p\frac{\partial f_{s0}^{(0)}}{\partial p}\right\} \quad (27)$$

The following step is to develop $\delta f_s(p,\theta,\varphi)$on the spherical harmonics $y_l^m(\theta,\varphi)$,**[8]**:

$$\delta f_s(p,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{m=l}\delta f_{sl}(p)\,y_l^m(\theta,\varphi) \quad (28)$$

The equation (25) then becomes:

$$\left(ik\frac{p_z}{m_e\gamma_l}cos\,\theta - C_{ei}(p)\right)\sum_{l=0}^{\infty}\sum_{m=-l}^{m=l}\delta f_{sl}(p)\,y_l^m(\theta,\varphi) = S_{E,1\pm1} + S_{B,2\pm1} \quad (29)$$

$$S_{E,1\pm1} = \pm\sqrt{\frac{2\pi}{3}}\, eE_s\, \frac{\partial f_{s0}^{(0)}}{\partial p} y_{1\pm1} \tag{30}$$

And

$$S_{B,2\pm1} = \mp\sqrt{\frac{2\pi}{15}}\frac{eB_s}{\gamma_l m_e} p_0^2 \left[\frac{1}{p^3}\frac{\partial}{\partial p}\left(p^3 \frac{\partial f_{s0}^{(0)}}{\partial p}\right) + \frac{1}{m_e^2 c^2 \gamma_l^2}(9 f_{s0}^{(0)} + \frac{7}{2} p.\frac{\partial f_{s0}^{(0)}}{\partial p})\right] y_2^{\pm1} \tag{31}$$

After some investigation using the properties of spherical harmonics, the equation (29) becomes:

$$C_{ei}(p)\delta f_{sl,m}(p) + \frac{ikp}{\gamma_l m_e}\left(\frac{l^2 - m^2}{4l^2 - 1}\right)\delta f_{sl-1,m}(p) + \frac{ikp}{\gamma_l m_e}\left(\frac{(l+1)^2 - m^2}{4(l+1)^2 - 1}\right)\delta f_{sl+1,m}(p) = S_{E,l}^m + S_{B,l}^m \tag{32}$$

where $S_{E,l}^m$ *and* $S_{B,l}^m$ are respectively the projections of S_E *and* S_B on the spherical harmonic of order (l,m). Note here that only the components $S_{E,1}^{\pm1}$ *and* $S_{B,2}^{\pm1}$ are not vanishing.

The equation (32) is the basic equation for calculate $f_s^{(1)}$ .It is a recurrence relation between $\delta f_{sl,m}$, $\delta f_{sl-1,m}$, and $\delta f_{sl+1,m}$. Note that for ≥ 3 , this equation can be write as:

$$C_{ei}(p)\delta f_{sl,m}(p) + \frac{ikp}{\gamma_l m_e}\left(\frac{l^2 - m^2}{4l^2 - 1}\right)\delta f_{sl-1,m}(p) + \frac{ikp}{\gamma_l m_e}\left(\frac{(l+1)^2 - m^2}{4(l+1)^2 - 1}\right)\delta f_{sl+1,m}(p) = 0 \tag{33}$$

The above system (33) is resolved by using a mathematical method based on the inversion of the collisions propagator in spherical harmonics basis using the continuous fractions **[10]**. The following solution is obtained:

$$\delta f_{s3,m} = -ik\frac{p^4}{\gamma_l^4 m_e^4}\left(\frac{9 - m^2}{35}\right) F_{3,m}\delta f_{s2,m} \tag{34}$$

where $F_{3,m}$ means the continuous fraction defined by the recurrence relation:

$$F_{l,m}(k,p) = \left[l(l+1)v + \frac{k^2 P^8}{\gamma_l^8 m_e^8}\left(\frac{(l+1)^2 - m^2}{4(l+1)^2 - 1}\right) F_{l+1,m}\right]^{-1} \tag{35}$$

Note here that the equation (34) is the exact solution of (32). It gives a relation between $\delta f_{s3,m}$ and $\delta f_{s2,m}$ with including the contributions of all anisotropies $\delta f_{sl,m}$ through the continuous fractions $F_{l,m}$.

From equations (32) and (34) coupled with the continuous fraction (35), we can calculate all anisotropic $\delta f_{sl,m}$ of the perturbed distribution function. In this work, we limit to $\delta f_{s1,1}$ and $\delta f_{s1,-1}$ enough for study the Weibel instability.

The equation of $\delta f_{s1,1}$, is deduced from (32) by putting $l = 1 \; and \; m = 1$, so:

$$v\, l(l+1)\delta f_{s1,1} + \sqrt{\frac{3}{15}} ik \frac{p^4}{\gamma_l^4 m_e^4} \delta f_{s2,1} = \frac{p^3}{\gamma_l^3 m_e^3} S_{E1,1} \tag{36}$$

The equation of $\delta f_{s2,1}$ is deduced also from (32) by putting $l = 2 \; and \; m = 1$, so:

$$v\, l(l+1)\delta f_{s2,1} + \sqrt{\frac{3}{15}} ik \frac{p^4}{\gamma_l^4 m_e^4} \delta f_{s1,1} + \sqrt{\frac{8}{35}} ik \frac{p^4}{\gamma_l^4 m_e^4} \delta f_{s3,1} = \frac{p^3}{\gamma_l^3 m_e^3} S_{B2,1} \tag{37}$$

With substitution of (34) in (37) and by using (35), starting from (35), we obtain after some algebra the explicit expression of $f_{s1,1}$, so:

$$\delta f_{s1,1} = \frac{i\sqrt{5}}{kp} \gamma_l m_e (1 - 2vF_{1,1}) S_{B2,1} + \frac{p^3}{\gamma_l^3 m_e^3} F_{1,1} S_{E1,1} \tag{38}$$

By the same method, we compute the explicit expression of $\delta f_{s1,-1}$, so:

$$\delta f_{s1,-1} = \frac{i\sqrt{5}}{kp} \gamma_l m_e (1 - 2vF_{1,-1}) S_{B2,-1} + \frac{p^3}{\gamma_l^3 m_e^3} F_{1,-1} S_{E1,-1} \tag{39}$$

by using the expression of $f_{s2}^{(0)}$ [equation (22)], the components $\delta f_{sl\pm1}$ use later is obtained as:

$$\delta f_{sl\pm1} = \pm \sqrt{\frac{2\pi}{3} \frac{eB_s}{k}} \omega F_{1,1} \frac{p^3}{\gamma_l^3 m_e^3} \frac{\partial f_{s0}^{(0)}}{\partial p}$$

$$\mp (1 + 2vF_{1,1}) \gamma_l m e B_s \sqrt{\frac{2\pi}{15}} p_0^2 \left[\frac{1}{p^3} \frac{\partial}{\partial p} \left(p^3 \frac{\partial f_{s0}^{(0)}}{\partial p} \right) + \frac{1}{m_e^2 c^2 \gamma_l^2} (9 f_{s0}^{(0)} + \frac{7}{2} p \frac{\partial f_{s0}^{(0)}}{\partial p}) \right] \tag{40}$$

4-relativistic Weibel instability analysis

This paragraph is devoted to the analysis growth rate of the relativistic Weibel instability. We determine the dispersion relation of the Weibel modes and deduce the growth rate of

Weibel instability; The dispersion relation of electromagnetic modes can be calculated using the perturbed Fokker-Planck equation coupled with Maxwell's equations presented as follows[11]:

$$\vec{\nabla} \times \vec{E_s} =- \frac{\partial \vec{B}_s}{\partial t} \tag{41}$$

and

$$\vec{\nabla} \times \vec{B_s} =- \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}_s}{\partial t} \tag{42}$$

is the current density defined by $\vec{j}$ where

$$\vec{j} =- i\mu_0 e \int \frac{p}{\gamma_l m_e} \delta f(\vec{r},\vec{p},t) \overrightarrow{dp} \tag{43}$$

By considering that the spatio-temporal dependence of the field $\vec{E}_s$ and $\vec{B_s}$ as a Fourier mode $\sim exp\left(i\omega t - i\vec{k}.\vec{r}\right)$ equations (41) and (43) can be represented as:

$$kE_s = \omega B_s \tag{44}$$

$$kB_s =- i\mu_0 e \int \frac{p}{\gamma_l m_e} \delta f_s \, d\vec{p} \tag{45}$$

By developing the function δf_s, In the spherical harmonics basis $y_l^m(\theta,\varphi)$, the equation (45) reads as:

$$kB_s =- ie\mu_0 \sqrt{\frac{2\pi}{3}} \int_0^\infty \left(\frac{p}{\gamma_l m_e}\right)^3 (\delta f_{s1.-1} - \delta f_{s1.1}) dp \tag{46}$$

for $f_{s0}^{(0)} = \mathrm{F(p)}$,Where $F(p)$ is The Jüttner (relativistic Maxwellian) distribution function**[12]** is given by:

$$\mathrm{F(p)} = A \exp\left(- E / \mathrm{k_b} T\right) \tag{47}$$

Where E is the energy of particle given by:

$$E = \mathrm{m_e c^2} (\gamma_l - 1) \tag{48}$$

$$A(\mu) = \frac{1}{\mathrm{K_2(\mu)}} \frac{\mu}{4\pi\,(\mathrm{m\,c})^3} \quad , \quad \mu =- \frac{\mathrm{m_e c^2}}{\mathrm{k_b T}} \tag{49}$$

$\gamma_l = \left(1 + \frac{p^2}{m_e^2 c^2}\right)^{1/2}$ is the Lorentz factor and $\mathrm{K_2(\mu)}$ denotes the modified Bessel function defined by:

$$\mathrm{K}_2(\mu) = \frac{1}{2}(2\pi m_e k_b T)^{-3/2} e^{-\mu} \tag{50}$$

Finally The Jüttner (relativistic Maxwellian) distribution function is given by:

$$\mathrm{F(p)} = \frac{1}{\mathrm{K}_2(\mu)} \frac{\mu}{4\pi\,(\mathrm{m\,c})^3} \exp\left[-\frac{\mathrm{m_e c^2}}{\mathrm{k_b T}}\left(\left(1+\frac{\mathrm{p^2}}{\mathrm{m_e^2 c^2}}\right)^{\frac{1}{2}} - 1\right)\right] \tag{51}$$

This function can be presented in the case of weakly relativistic plasma, where $\eta \gg 1$ and the modified Bessel function can be written as : $\mathrm{K}_2(\eta) \approx \left(\frac{\pi}{2\eta}\right)^{\frac{1}{2}} \exp(-\eta)$. This approximation is justified in the inertial fusion experiments. Typically $T_e/m_e c^2 \simeq 0.02$ for $T_e = 10\,KeV$, By developing the relativistic Maxwellian distribution function as:

$$\mathrm{F(P)} = \frac{1}{2}(2\pi m_e k_b T)^{-3/2} \exp\left(-\frac{m_e c^2}{k_b T}\left(1+\frac{p^2}{2m_e^2 c^2}\right)\right) \tag{52}$$

Using the equation (40) (46) and (52) The dispersion relation in a plasma hight relativistic temperature, is obtained:

$$\frac{k^2c^2}{\omega_p^2} = i\frac{1}{3}\sqrt{\frac{2}{\pi}}\frac{1}{V_t^5}\,\omega \int_0^\infty \frac{p^7}{m_e^5\gamma_l^6} F_{1,1} \exp\left(-\frac{E}{k_b T}\right) dp$$
$$+ \frac{\sqrt{2}}{15\sqrt{3\pi}}\frac{p_0^2}{V_t^3}\int_0^\infty \left\{\frac{p^4}{m_e^6 c^2}\frac{1}{\gamma_l^3}\left[\frac{5}{2}-\frac{c^2}{V_t^2}\right] - \frac{p^2}{m_e^4\gamma_l^4}\left[\frac{9}{\gamma_l} - 4\frac{c^2}{V_t^2}\right]\right\}(1-2\nu F_{1,1}) \exp\left(-\frac{E}{k_b T}\right) d\,p \tag{53}$$

The relativistic growth rate of Weibel instable mode γ is obtained explicitly from this dispersion relation, so:

$$\gamma(k) = -\frac{3}{2}\sqrt{\frac{\pi}{2}} V_t^5 \frac{k^2c^2}{\omega_p^2} \frac{1}{\int_0^\infty \frac{p^7}{m_e^5\gamma_l^6} F_{1,1}\exp\left(-\frac{E}{k_bT}\right) dp}$$
$$+ \frac{m_e^2 V_t^4 p_0^2}{10\sqrt{3}\, p_t^2} \frac{\int_0^\infty \left\{\frac{p^4}{m_e^4}\frac{1}{\gamma_l}\left[\frac{5}{2}-\frac{c^2}{V_t^2}\right] - \frac{p^2}{m_e^2\gamma_l}\left[\frac{9}{\gamma_l} - 4\frac{c^2}{V_t^2}\right]\right\}(1-2\nu F_{1,1})\exp\left(-\frac{E}{k_bT}\right) d\,p}{\int_0^\infty \frac{p^7}{m_e^5\gamma_l^6} F_{1,1}\exp\left(-\frac{E}{k_bT}\right) dp} \tag{54}$$

Where

$$F_{1,1} = \frac{1}{\left(2\nu + \frac{1}{30\nu}\frac{k^2p^8}{m_e^8\gamma_l^8}\right)}$$

5- Discussion and Conclusion

The first term of equation (54) corresponds to a loss term due to Landau damping and to collisions effect; it is dominated by collisions loss in the limit: $(k\lambda_{ei} \ll 1)$ while in the non-collisional limit $(k\lambda_{ei} \gg 1)$, it is dominated by the Landau damping of electromagnetic modes.

The second term, $\sim p_0^2$, corresponds to the WI source. Equation (54) gives explicitly the growth rate of the Weibel modes excited by IB absorption in laser fusion plasma as function of laser pulse and plasma parameters via an integral form.

The spectra of the growth rate $\gamma(k)$ which give the growth rate of the all the instables Weibel modes (not only the γ_{max}). The calculated $\gamma(k)$ in our paper, contains two contributions: a Landau damping and an instability source propotional to the second ansisotropy of the distribution function developped on the legendre polynomials, f_2 which is propotional to the laser intensity via the term $p_0^2 \sim I$.

This shows clearly that the source of the anisotropy and consequentely of the instability is the laser heating.

We have presented in (Figures 1) the growth rate spectra of Weibel instability $\gamma(k)$, as function of the collision parameter $k\lambda_{ei}$ for typical parameters of the laser pulse and plasma. We point out that without the stabilization term, $S_{IB}\left(f_{s0}^{(0)}\right)$. In addition, the comparison of the obtained spectra with previous works shows an overestimates by two orders in the growth rate of the most unstable Weibel mode in the non-relativistic case.

In conclusion, the Weibel instability is theoretically studied using the Vlasov equation by considering the Krook collisions model. The dispersion relation of the Weibel modes is explicitly established under some justified approximation in the laser-fusion experiments [**13, 14**]**.** Taking into account to stabilization effect by the inclusion of the term $S_{IB}\left(f_s^{(0)}\right)$ led to a significant reduction in the Weibel instability growth rate. Numerical treatment of model equations shows that the growth rate of the most unstable Weibel mode decreases by two orders of magnitude. This decrease in the growth rate magnitude is accompanied by a greater reduction in the spectral range of instability. For high density plasma, the Weibel modes become completely stables. Therefore, the generation of magnetic fields by the Weibel instability due to inverse bremsstrahlung should not affect the experiences of inertial confinement fusion. Several possible extension of this study is possible; namely the taking into account of the nonlinear effect **[15, 16, 17]** due to the high intense laser pulse.

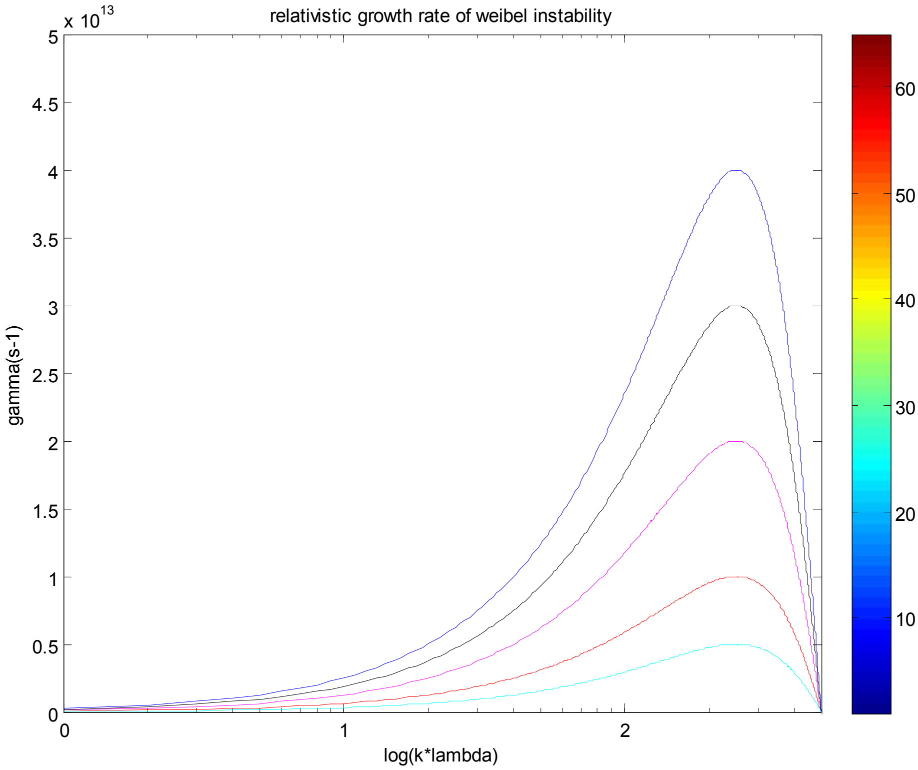

Figure 1: growth Rate of instability γ from the Krook model ,depending on the collision parameter $k\lambda_{ei}$, for typical parameters of laser pulse and fusion plasma: $T_e = 10KeV$, $\lambda_{ei} = 1\mu m$, $\lambda_l = 1.06\mu m$ $n_e=10^{26}$ cm^{-3} and p_0/p_t=0,1.

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